APPLICATION OF MULTIWAY DATA ANALYSIS IN CIVIL INFRASTRUCTURE SYSTEMS

- The National Bridge Inventory (NBI)

by

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Outline

- Introduction
- Basic Concepts in Multiway Data Analysis
- NBI applications
- Conclusion
- Future work

Conventional Data Analysis

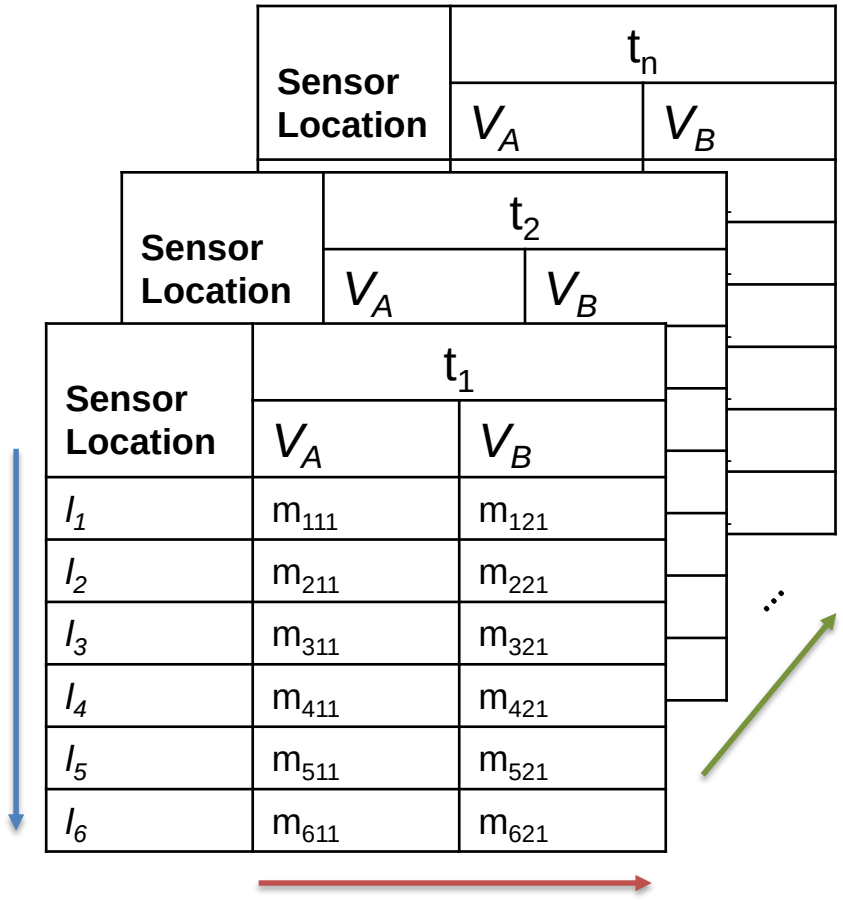
- Structured data
- Tables with rows and columns (Matrices)

Sensor Location	t_1	
	V_A	V_B
l_1	m_{11}	m_{12}
l_2	m_{21}	m_{22}
l_3	m_{31}	m_{32}
l_4	m_{41}	m_{42}
l_5	m_{51}	m_{52}
l_6	m_{61}	m_{62}

Multiway Data is Everywhere

Sensor Location	t_1	
	V_A	V_B
I_1	m_{11}	m_{12}
I_2	m_{21}	m_{22}
I_3	m_{31}	m_{32}
I_4	m_{41}	m_{42}
I_5	m_{51}	m_{52}
I_6	m_{61}	m_{62}

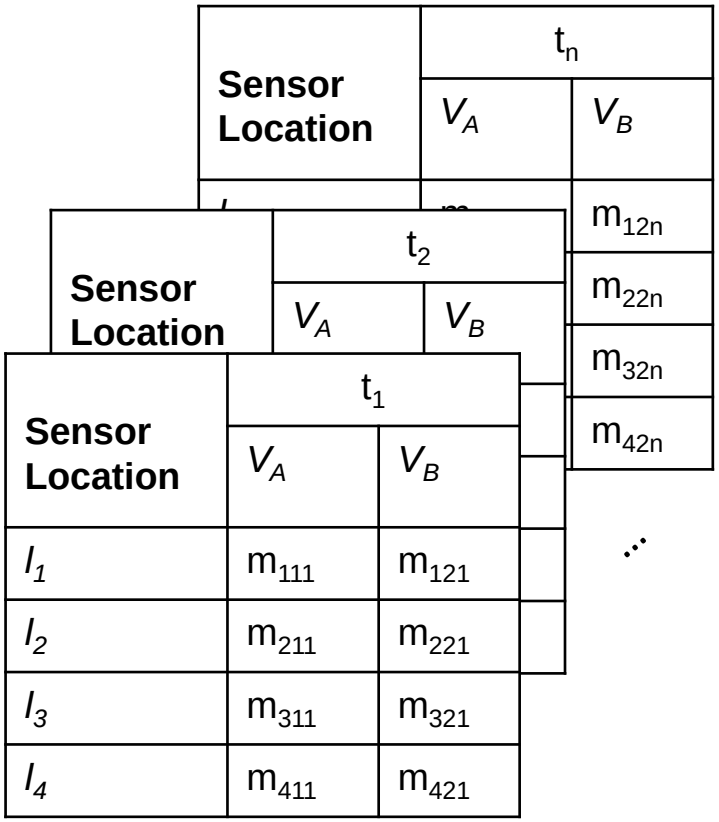
Two-way array



Three-way array

Why Multiway Data Analysis?

- Potential for loss of information using matrix analysis



Averaging across time

Sensor Location	$\overline{V_A}$	$\overline{V_B}$
l_1	$\bar{m}_{11}$	$\bar{m}_{12}$
l_2	$\bar{m}_{21}$	$\bar{m}_{22}$
l_3	$\bar{m}_{31}$	$\bar{m}_{32}$
l_4	$\bar{m}_{41}$	$\bar{m}_{42}$

May lose information on temporal variation

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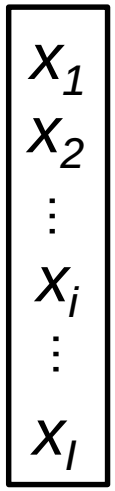
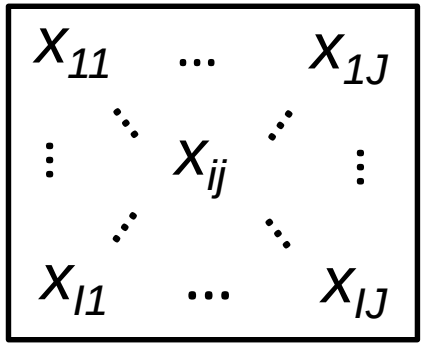
Definition of Multiway Data

- Multiway arrays are also known as tensors
- Higher-order generalizations of scalars, vectors and matrices
- N-way tensors



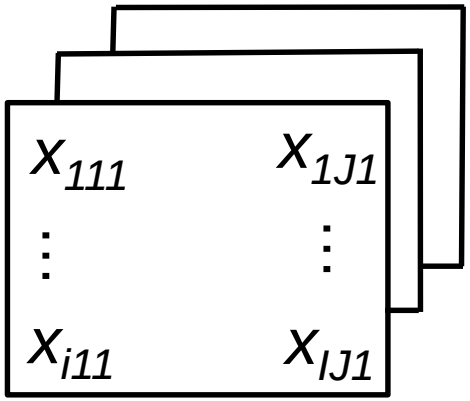
$\boldsymbol{x} \in \mathbb{R}^I$

Vector
(One-way tensor)



$\boldsymbol{X} \in \mathbb{R}^{I \times J}$

Matrix
(Two-way tensor)

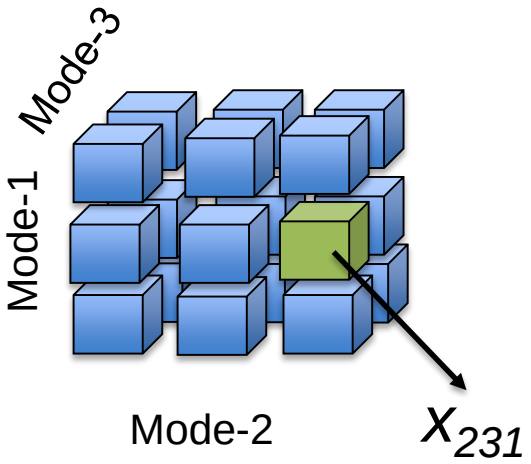


$\boldsymbol{X} \in \mathbb{R}^{I \times J \times K}$

(Three-way tensor)

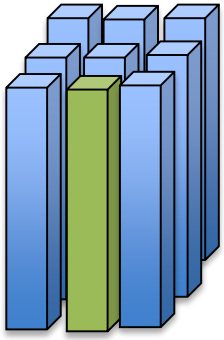
Definition of Multiway arrays (2)

Typical 3D dataset

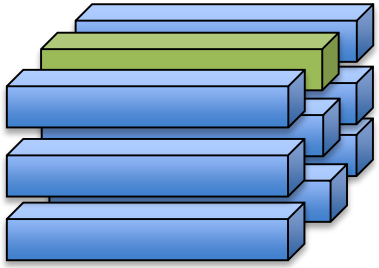


$\underline{X}$

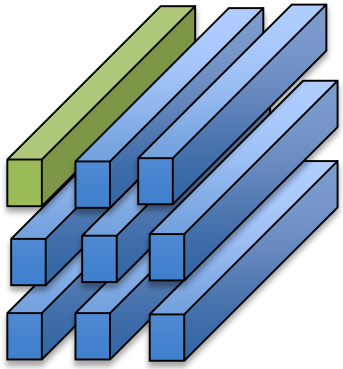
Subarrays



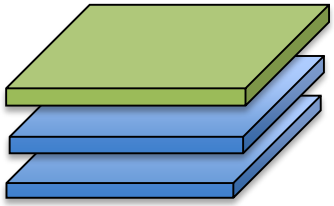
Mode-1 Fiber
$x_{:21}$



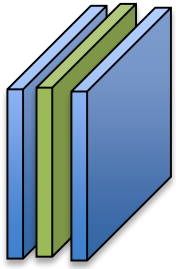
Mode-2 Fiber
$x_{1:2}$



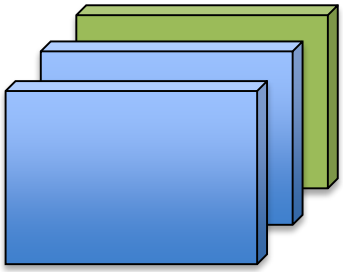
Mode-3 Fiber
$X_{11:}$



Horizontal slice
$X_{1::}$



Lateral slice
$X_{:2:}$



Frontal slice
$X_{::3}$

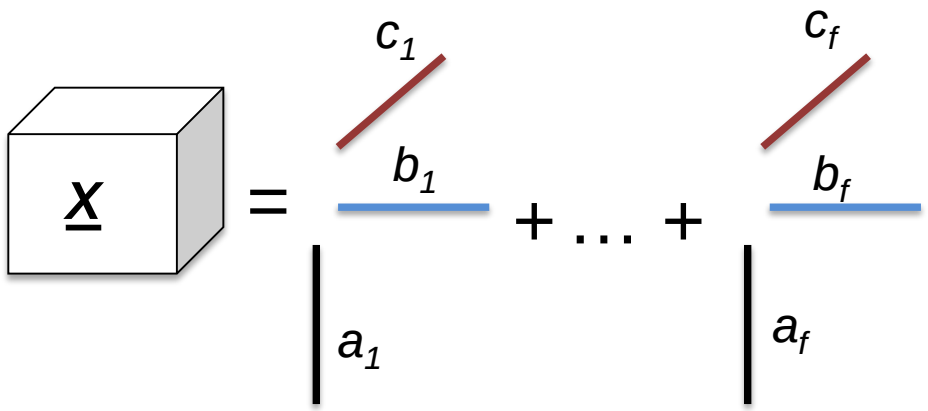
Tensor Decomposition

- Reducing data to lower-order forms for analysis (classification, prediction, clustering)

- 2 main approaches
  - Canonical decomposition/Parallel Factors (CANDECOMP/PARAFAC; **CP decomposition**)
  - **Tucker decomposition**

Tensor Decomposition

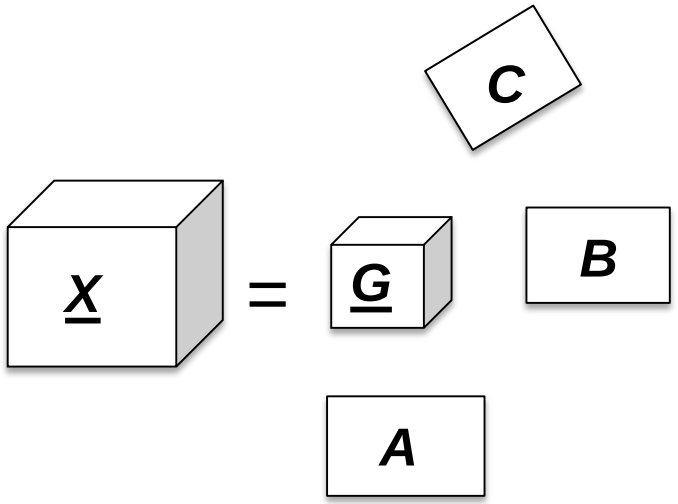
CP Decomposition



$$\underline{\boldsymbol{X}} \approx \sum_{f=1}^{F} \boldsymbol{a_f} \circ \boldsymbol{b_f} \circ \boldsymbol{c_f}$$

where $\underline{\boldsymbol{X}} \in \mathbb{R}^{I \times J \times K}$, $f < I, J$ and K
$\boldsymbol{a_f}$, $\boldsymbol{b_f}$ and $\boldsymbol{c_f}$= fth loading factor in modes $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively &
$\circ$ = the outer product of arrays

Tucker Decomposition



$$\underline{\boldsymbol{X}} \approx \underline{\boldsymbol{G}} \times_L \boldsymbol{A} \times_M \boldsymbol{B} \times_N \boldsymbol{C}$$

where $\underline{\boldsymbol{X}} \in \mathbb{R}^{I \times J \times K}, L, M, N \ll I, J, K$
$\underline{\boldsymbol{G}} \in \mathbb{R}^{L \times M \times N}$, $\boldsymbol{A} \in \mathbb{R}^{I \times L}$, $\boldsymbol{B} \in \mathbb{R}^{J \times M}$, and
$\boldsymbol{C} \in \mathbb{R}^{K \times N}$

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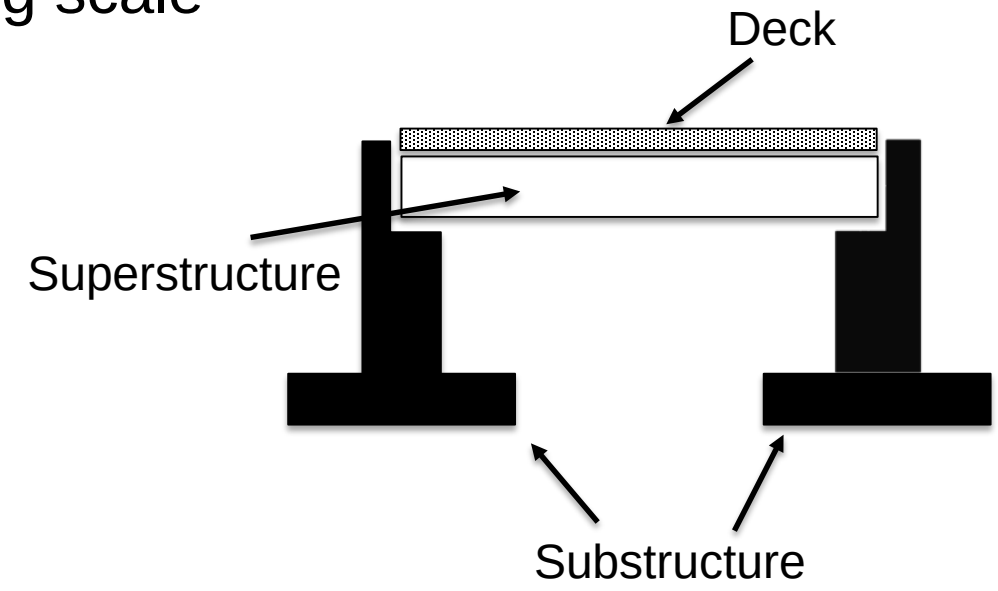
National Bridge Inventory (NBI)

- Largest bridge database in the world
- Information on condition, location and other attributes of >600, 000 bridges
- Ratings assigned to bridge components
- *0-9* (failed-excellent) rating scale
- Bridge Status:
  - Structurally deficient
  - Functionally obsolete
  - Not deficient

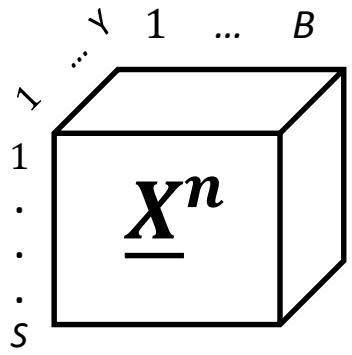


Aim of Analysis

- Use the NBI as a basis for prediction of bridge structural deficiency in the US.
- 6 Superstructural types
  - Slab
  - Stringer/ Multibeam
  - Girder & Floorbeam
  - Tee Beam
  - Box beam (multiple)
  - Arch-deck
- Hybrid approach
- Tensor Decomposition & Simple Exponential Smoothing (SES)

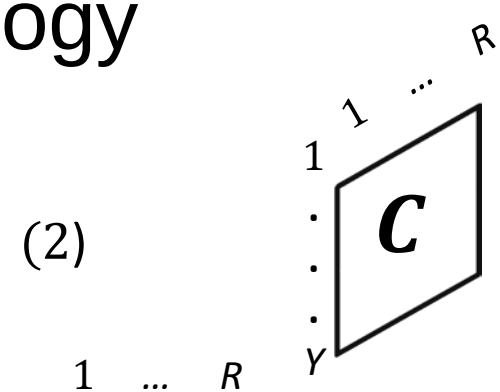
Methodology

(1)

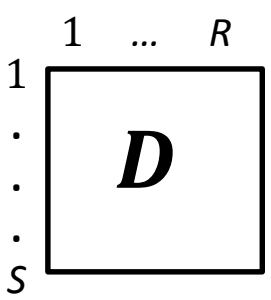


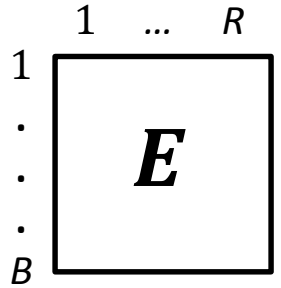
Tensor Decomposition

(2)

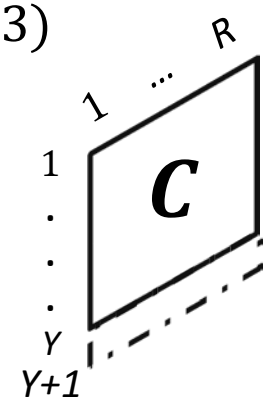


Loading Matrix for Time Mode





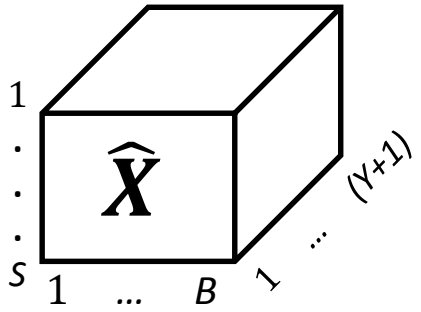
(3)



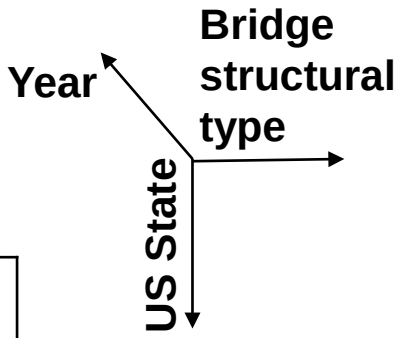
Time Series (SES) Modeling to determine Y+1 elements

Forming New Array Including successive time step

(4)



Methodology (1)

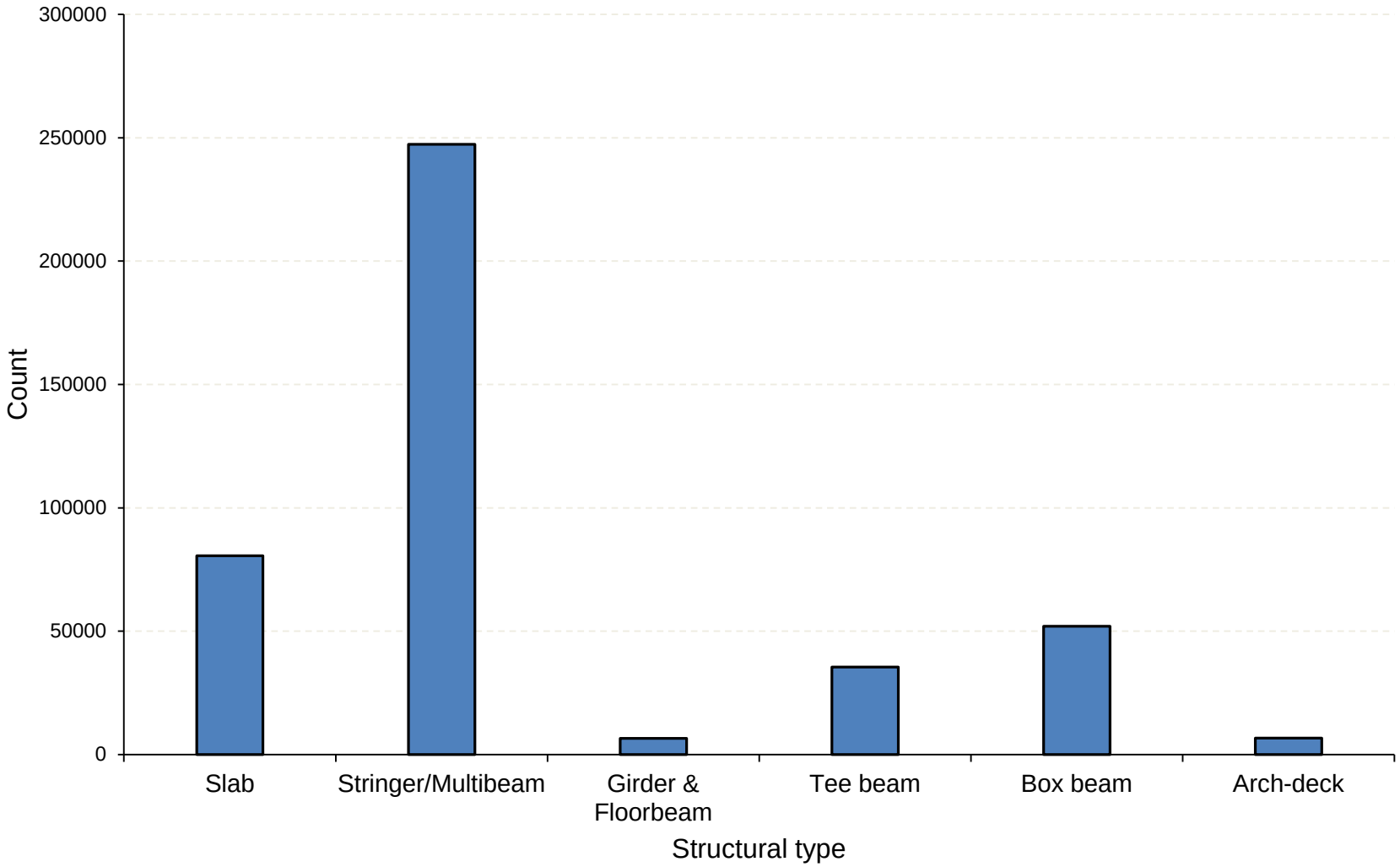


- $\underline{\boldsymbol{X}} \in \mathbb{R}^{52\times6\times22}$
- $\underline{\boldsymbol{X}}^{\boldsymbol{n}}$ → Normalization with Structural deficiency ratio

Structural deficiency (SD) ratio= $\frac{\boldsymbol{n_{s,b,y}}}{\boldsymbol{N_{s,b,y}}}$

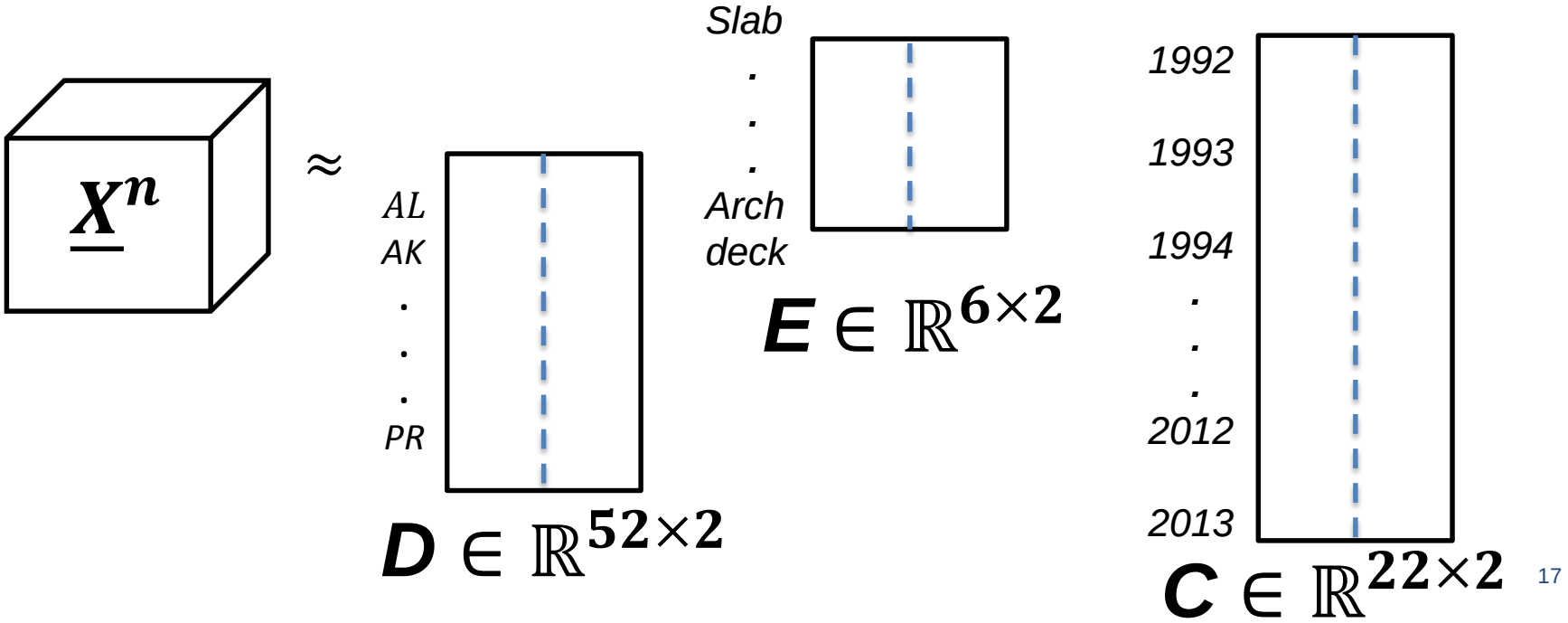
$n_{s,b,y}$= no. of structurally deficient bridges in state *s*, of structural type *b* in year *y*
$N_{s,b,y}$ =total no. of bridges in state *s* of structural type *b* in year *y*

Total bridge counts of the 6 superstructure types (2013)

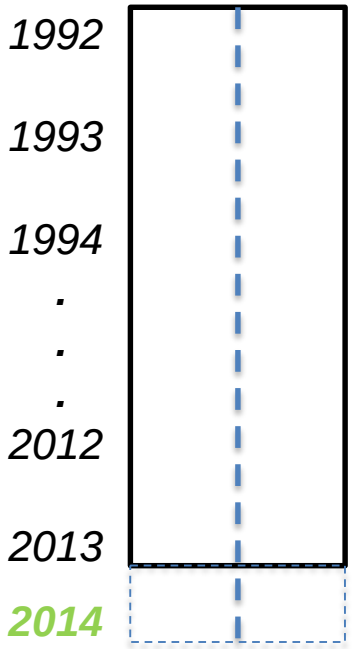


Methodology (2)

- 2-component CP model is fitted
- Nonnegativity constraints imposed on components of model
- Lower core-consistency diagnostic (CORCONDIAG)= *67.3414%*



Methodology (3)

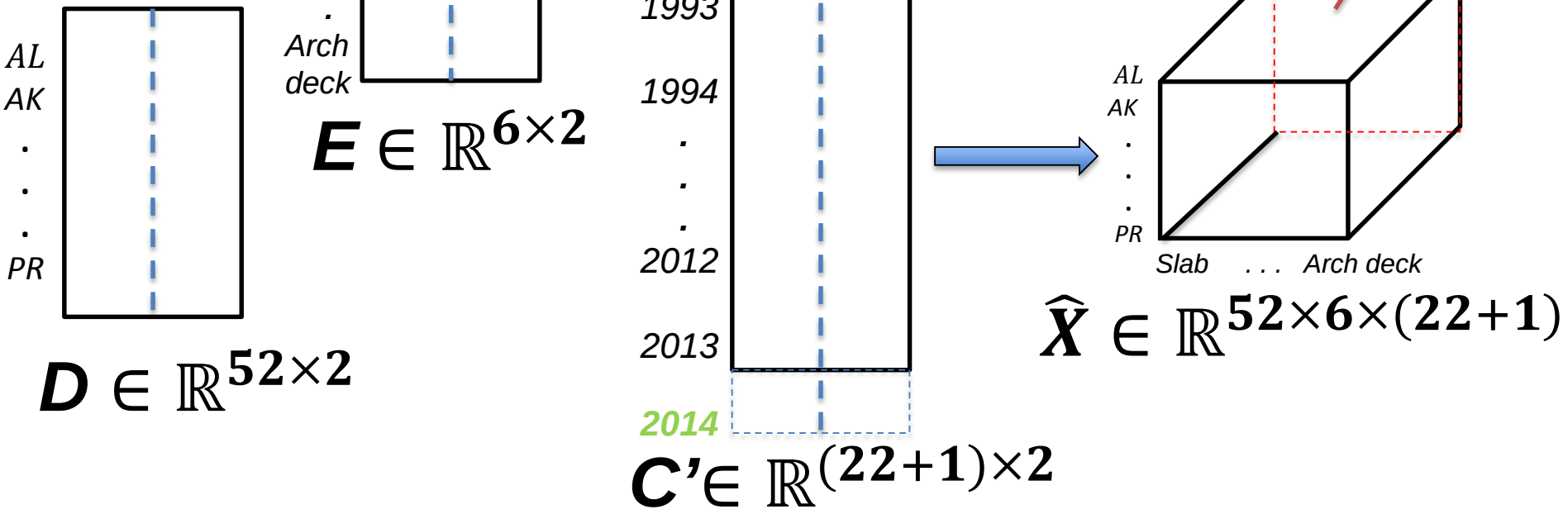


$C' \in \mathbb{R}^{(22+1)\times 2}$

- Simple Exponential Smoothing
- $\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1}$

where $\hat{y}_{t+1|t}$=forecast for time *t+1* given based on t time steps;
y_t= observation for time t;
$\hat{y}_{t|t-1}$= forecasted value for time *t* based on *t-1* time steps; and
α= smoothing factor $(0 < \alpha < 1)$.

Methodology (4)



$\boldsymbol{D} \in \mathbb{R}^{52\times 2}$

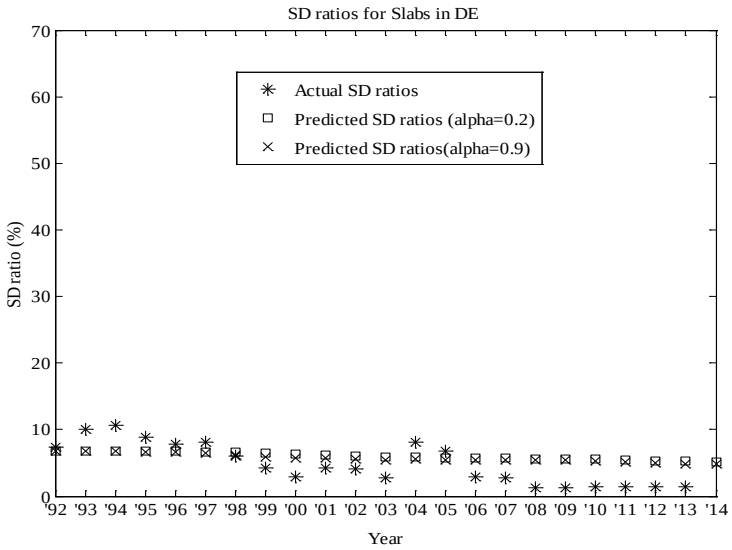
$\boldsymbol{E} \in \mathbb{R}^{6\times 2}$

$\boldsymbol{C'} \in \mathbb{R}^{(22+1)\times 2}$

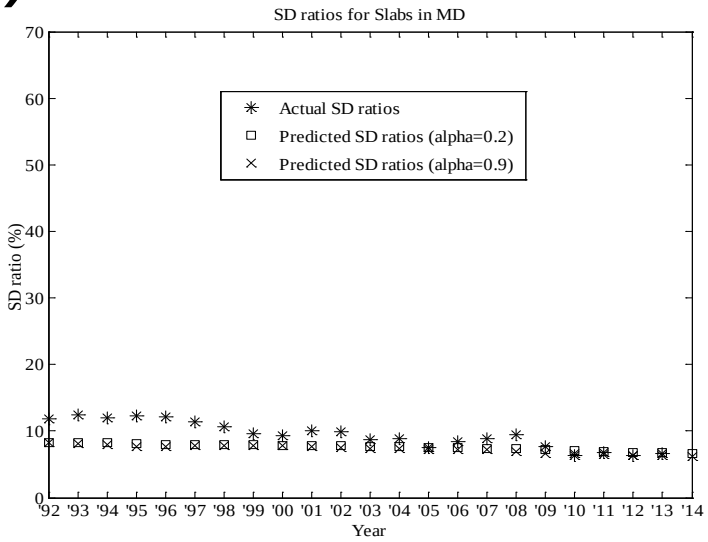
$\hat{\boldsymbol{X}} \in \mathbb{R}^{52\times 6\times (22+1)}$

- Isolate new slice for predicted values for *2014*
- State and Structural type prediction values can be viewed simultaneously

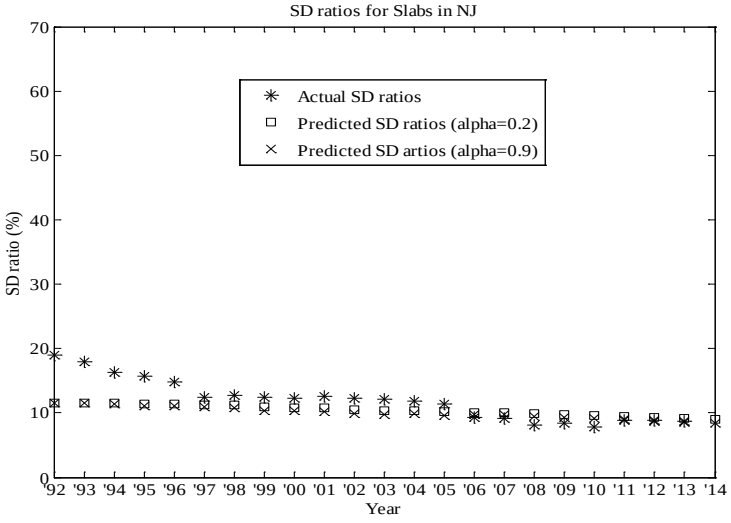
Results (1)- Slabs



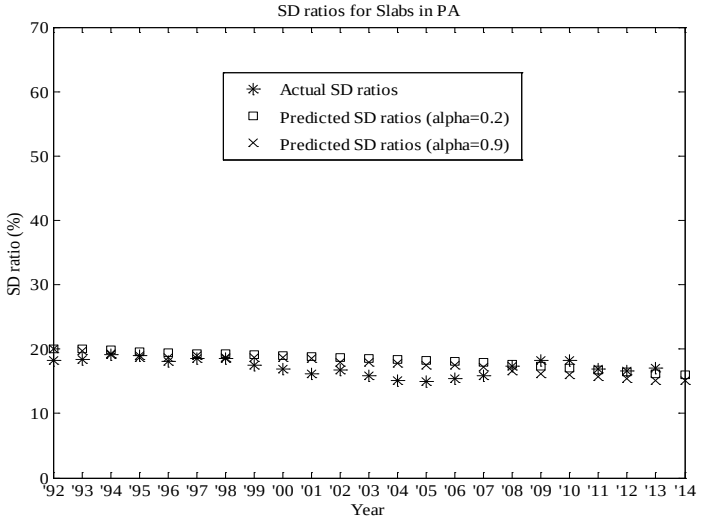
Delaware



Maryland

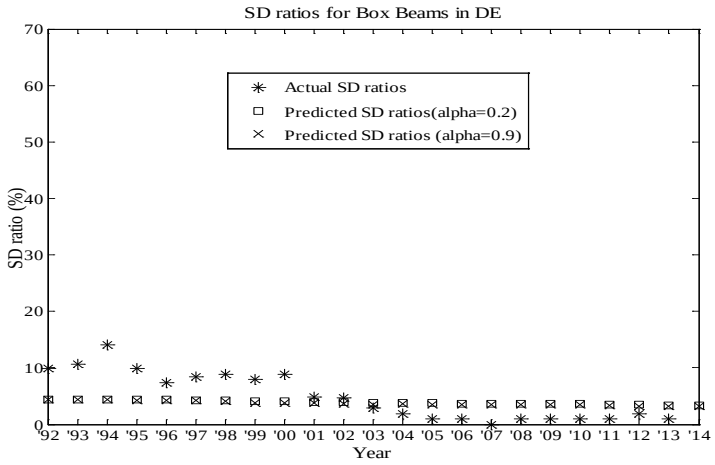


New Jersey

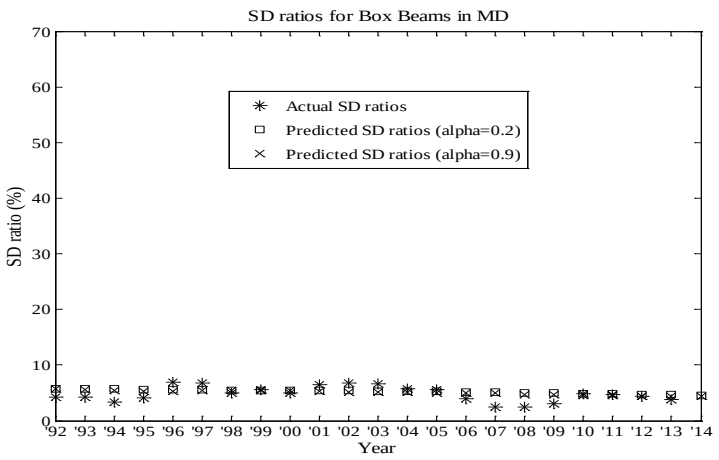


Pennsylvania

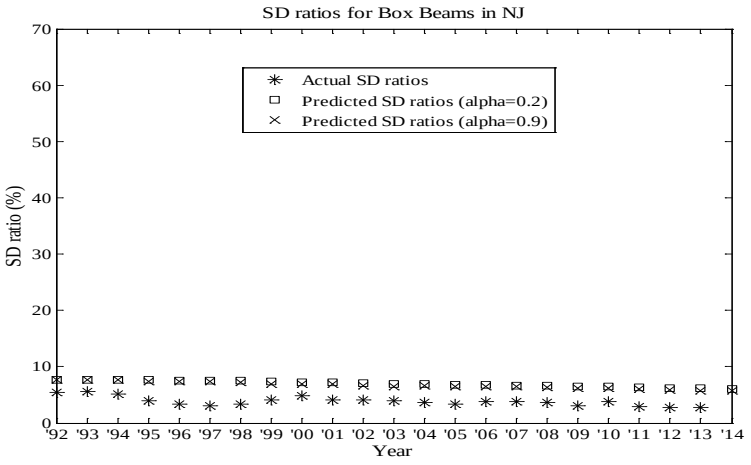
Results (2)- Box beams



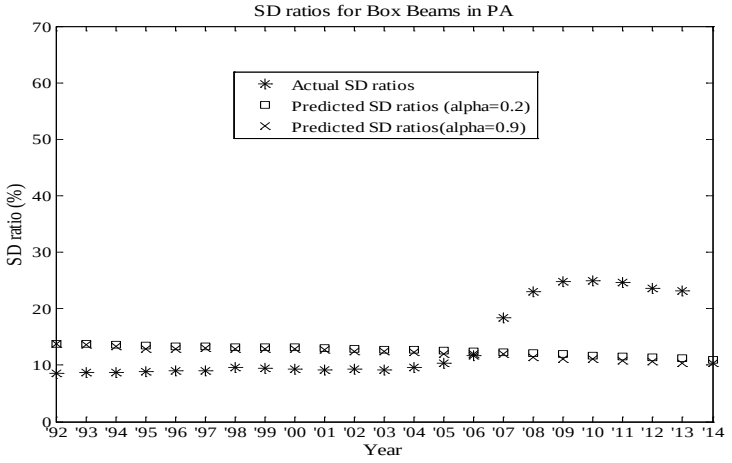
Delaware



Maryland



New Jersey



Pennsylvania

Mean Absolute Errors (MAE) for predicted SD ratios

Bridge Type	$\alpha = 0.2$	$\alpha = 0.9$
Delaware		
Slab	2.685	2.509
Girder & Floorbeam	14.473	14.631
Box Beam	3.455	3.442
Pennsylvania		
Slab	1.518	1.425
Girder & Floorbeam	7.107	5.078
Box Beam	6.233	6.222
New Jersey		
Slab	2.177	2.279
Girder & Floorbeam	4.176	2.390
Box Beam	3.172	2.893
Maryland		
Slab	1.924	2.087
Girder & Floorbeam	4.428	4.069
Box Beam	1.131	1.104

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Conclusion

- Tensor decomposition provides an alternative bridge condition methodology
- SD ratios for all states and superstructure types under consideration can be viewed simultaneously
- Results encourage further studies of multiway data analysis techniques in civil infrastructure systems

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Future Work

- Focusing on specific bridges within US states
- Using other time series prediction techniques
- Integrating other time-varying parameters into analysis